Do Mutation Rates Match the Kelly Criterion?

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The Kelly Criterion defines an optimal betting strategy for games that have a defined risk and payoff. It was developed by John Kelly, Jr. at Bell Labs (Kelly, 1956). Given a bet with a probability of success P and a payout of B, the Kelly criterion tells you the size of your bet compared to your overall bankroll. The Kelly criterion is given as

$$\frac{PB + P - 1}{B} \tag{1}$$

This formula can be derived from a formula for an expected total payoff of the bet given by the equation

$$T = A(1 + Bf)^{NP}(1 - f)^{N(1-P)} \tag{2}$$

where T is the total winnings, A is your starting amount, N is the number of trials, and f is the bet size. Optimizing for T yields Equation 1.

Many researchers have discussed the concept of mutations in populations as "bet hedging." (Philippi and Seger, 1989; Bartlett, 2008; Simons, 2011; Grimbergen et al., 2015) Since the Kelly criterion allows one to at least theoretically calculate the optimum bet size for each configuration, it might be possible to calculate various optimum mutation rates at different sites and compare them to their optimal size according to the Kelly criterion, or an adjusted version of it.

Most analysis of bet hedging has merely checked to see if the hedging strategy is empirically beneficial (Childs, Metcalf, and Rees, 2010; Simons, 2011) or potentially evolvable (King and Masel, 2007). Applying the Kelly criterion may be able to help determine how optimal organisms' various bet hedging strategies are.

One possible experimental approach would be to provide organisms with a long-term, continually-varying environment. After many generations, it would be interesting to check if the mutation rates for adaptive switching between environments had any relation to the theoretical considerations of the Kelly criterion, or any other theoretical hedging system.

Bartlett, J L (2008). "Statistical and Philosophical Notions of Randomness in Creation Biology". In: *Creation Research Society Quarterly* 45, pp. 91–99.

Childs, D Z, C J E Metcalf, and M Rees (2010). "Evolutionary bet-hedging in the real world: empirical evidence and challenges revealed by plants". In: *Proceedings fo the Royal Society B: Biological Sciences* 277, pp. 3055–3064. DOI: `10.1098/rspb.2010.0707`.

Grimbergen, A J et al. (2015). "Microbial bet-hedging: the power of being different". In: *Current Opinion in Microbiology* 25, pp. 67–72.

Kelly, J L (1956). "A New Interpretation of Information Rate". In: *The Bell System Technical Journal* 35, pp. 917–926. URL: `http://www.herrold.com/brokerage/kelly.pdf`.

King, O D and J Masel (2007). "The evolution of bet-hedging adaptations to rare scenarios". In: *Theoretical Population Biology* 72, pp. 560–575. DOI: `10.1016/j.tpb.2007.08.006`.

Philippi, T and J Seger (1989). "Hedging One's Evolutionary Bets, Revisited". In: *Trends in Ecology and Evolution* 4.2, pp. 41–44.

Simons, A M (2011). "Modes of response to environmental change and the elusive empirical evidence for bet hedging". In: *Proceedings of the Royal Society B: Biological Sciences* 278.1712, pp. 1601–1609. DOI: `10.1098/rspb.2011.0176`. URL: `https://royalsocietypublishing.org/doi/10.1098/rspb.2011.0176`.

On the Logic of Being and Wigner's Astonishment Regarding the Applicability of Mathematics

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The Nobel Prize winning Physicist, Eugene Wigner, famously posed a powerful challenge (1960) by asking why is mathematics so effective, especially in the physical sciences (Wigner, 1960). It is possible that the reason for the effectiveness of mathematics is not because mathematics is in any way causative, but instead because mathematics studies the structure of logical possibility and constraint. When plugged into a possible world, mathematics gives us the tools to analyze the logically possible outcomes. Therefore, when a possible world that is expressed mathematically sufficiently aligns with reality, mathematics becomes effective at expressing relationships and outcomes.

For example, beings (as well as possible beings and things impossible of being) can be understood in the context of possible worlds. A "possible world" is a sufficiently complete description of possible states of affairs described through chains of propositions. We may observe that things impossible of being, such as a square circle, have in them mutually inconsistent required core attributes; they cannot be realised in any possible world. Possible beings would exist in at least one possible world were it actualised.

For instance, a contingent being B that depends on C might exist in a world W and not in a closely neighbouring one W' if C is present in W but not W'; C thus being an enabling, necessary causal factor for B. By contrast, a necessary being F will exist in all possible worlds, showing itself to be a framework element for such a world.

A key insight is that for any world W to be distinct from W' it requires some factor A in W that is absent in W'. We may then partition the factors of W as $W = \{A|\neg A\}$. After partitioning, we will have two distinct groups—the factor A and all of the factors which are not A. The null set corresponds to zero. Each particular set in the partition can be counted as the number one, and the combination of both partitions (even in a single world where A is an empty set) is two. Thus, for any particular possible world W, the quantities 0, 1, 2 are necessarily present. Taking the von Neumann construction, immediately we find $\mathbb{N}$, thence (using additive inverses) $\mathbb{Z}$, so also (taking ratios) $\mathbb{Q}$ and (summing convergent power series) $\mathbb{R}$; where $\mathbb{Z}$ provides unit-stepped mileposts in $\mathbb{R}$. That is, a structured core of quantities will be present in any W, and we may regard mathematics as the study of the logic of structure and quantity. Extensions to the hyperreals $\mathbb{R}^*$ follow by construction of some H that has as reciprocal $h = \frac{1}{H}$ closer to 0 than $\frac{1}{n}$ for any n in $\mathbb{N}$. Therefore, relationships and linked operations across such quantities will also be present, or may be constructed as needed. Illustrating, after Abraham Robinson (Robinson, 1966), hyperreals allow calculus to be treated as extensions of algebra in $\mathbb{R}^*$.

Thus, while bare distinct identity and coherence focused on quantities will not *cause* things by the inherent potential or action of such entities, they instead are logical constraints on being and are tied to what can or must be or cannot be or happens not to be. So, too, we may see that the abstract logic model worlds that we may construct then lead to key entities that if necessary are framework to any possible world; thus applicable to our common world. By contrast, if certain quantities and relationships are merely part of the contingencies of some W'' that is close enough to our own, they may provide adequate analogies for modelling.

As a result, we have good reason to expect that mathematical reasoning and core entities will in many cases be highly relevant to and have powerful predictive power for our common world.

Robinson, Abraham (1966). *Non-Standard Analysis.* Amsterdam: North-Holland Publishing Company.

Wigner, Eugene (1960). "The Unreasonable Effectiveness of Mathematics in the Natural Sciences". In: *Communications in Pure and Applied Mathematics* 13.1, pp. 1–14. DOI: 10.1002/cpa.3160130102.

Independence Conservation and Evolutionary Algorithms

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Levin's Law of Independence Conservation

Leonid Levin's 1984 article (Levin, 1984) is the first to this author's knowledge to prove a fully stochastic conservation of information law. Levin titled his law 'independence conservation' which he considered fairly obvious, describing it as "Torturing an uninformed witness cannot give information about the crime!"

Levin's law is not well known, which is unfortunate since the more commonly known conservation laws are focused either only on the random or deterministic case. Levin's law is remarkable because it unifies both the random and deterministic cases, showing that the combination also cannot result in information increase.

The second remarkable thing about his law is how easy it is to prove, given some preliminaries about algorithmic information.